ATS 602 – Homework 3

due Friday, March 10 (by Noon)

1. **QG dynamics in log-pressure coordinates.**

   (a) Fill in the steps to derive the PV equation in the QG log-pressure coordinates (at the bottom of page 9 of the notes). Explain why the thermodynamic equation in this QG system becomes $D_g\theta'/Dt + W_a\partial_Z\theta_{\text{ref}} = 0$.

   (b) Derive the following thermal wind balance relation in these coordinates:

   $$\partial_Z u_g = -\frac{g\partial_y\theta}{f_0\theta_{\text{ref}}}$$

   Hint: show that the scale height can be expressed as $H = RT_{\text{ref}}/g$. You also need to convince yourself that in these coordinates $\partial_y\theta/\theta_{\text{ref}} = \partial_y T/T_{\text{ref}}$.

   (c) Define a basic state flow: $U = U(y, Z)$ and $V = 0$, with a basic state potential temperature Θ in thermal wind balance with this flow. Further, define the isentropic slope as $s_\Theta \equiv dZ/dy|_\Theta = -\partial_y\Theta/\partial_Z\theta_{\text{ref}}$. Then show that the resulting basic state PV gradient can be written as:

   $$\partial_y Q = \beta - \partial_{yy}U - f_0\partial_Z s_\Theta$$

   (d) Estimate the rough size of each term in above expression for $\partial_y Q$ near the extratropical tropopause, based on the average cross sections shown in the overview slides from class 1. Briefly interpret/discuss.

2. **Two-Layer Shallow Water QG Rossby Waves.** Consider the two-layer problem, formulated in terms of QG PV (see notes from February 23):

$$\frac{D_1 \tilde{q}_1}{Dt} = 0\,, \quad \frac{D_2 \tilde{q}_2}{Dt} = 0\,, \qquad \frac{D_j}{Dt} \equiv \partial_t - \partial_y \psi_j \partial_x + \partial_x \psi_j \partial_y\,, \quad j = 1, 2$$

$$\text{with:} \quad \tilde{q}_1 = \beta y + \vec{\nabla}^2 \psi_1 - \frac{\psi_1 - \psi_2}{{L'_d}^2}\,, \qquad \tilde{q}_2 = \beta y + \vec{\nabla}^2 \psi_2 - \frac{\psi_1}{L_d^2} + \frac{\psi_1 - \psi_2}{{L'_d}^2}\,,$$

where $L_d^2 \equiv gH/f_0^2$, ${L'_d}^2 \equiv g'_{12} H/f_0^2$, and for simplicity we assume $H = H_1 = H_2$.

(a) Linearize the problem around a basic state of uniform flow in the x-direction: $u_j = U + u'_j$ (with $U =$ const.), $v_j = v'_j$, etc., $j = 1, 2$ (this is similar to the QG Rossby wave problem in a single layer).

(b) Show that the barotropic solution, i.e. where $\psi_1 = \psi_2$ (no vertical structure), only exists if the third term in $\tilde{q}_2$ above is neglected (i.e. $|\psi_1/L_d^2| \ll |(\psi_1 - \psi_2)/{L'_d}^2|$; this corresponds to a rigid lid approximation with negligible perturbations in η_0). Interpret this physically.

(c) By making the rigid lid approximation (see above), solve the linearized problem for Rossby waves, assuming solutions of the form $\psi_j = A_j \exp[i(kx + ly - \omega t)]$, $j = 1, 2$. *Hint:* There are two types of solutions with different dispersion relations – a barotropic solution ($A_1 = A_2$) and a baroclinic solution ($A_1 \neq A_2$). Briefly discuss your dispersion relations, comparing them to the divergent and non-divergent single layer versions.