ATS 602 – Homework 2

due Tuesday, February 21

1. **More Hoskins et al. PV paper.**

   (a) Recreate Figures 9a and 14 (feel free to pick other examples of upper level cyclones and anticyclones from the data). You need the pressure level data for this (note that ECMWF provides PV on those levels).

   (b) Work through section 3 of the paper, in particular up to Eq. 29. The material toward the end of this section may be skimmed. Write down a few questions for class discussion. Briefly discuss the idealized cross-sections (Figure 15) in relation to the ones obtained from the data under (a).

   (c) Read through sections 5a and 6a; if you have any questions, write them down.

2. **Shallow water Rossby waves and PV on isentropic surfaces.** Consider the dispersion relation for shallow water divergent (QG) Rossby waves:

$$\omega = Uk - k\frac{\beta + UL_d^{-2}}{K^2 + L_d^{-2}}$$

$$\Rightarrow \quad c_x = \frac{\omega}{k} = U - \frac{\beta + UL_d^{-2}}{K^2 + L_d^{-2}}\,, \quad c_{g,x} = \partial_k \omega = U + (\beta + UL_d^{-2})\frac{k^2 - l^2 - L_d^{-2}}{(K^2 + L_d^{-2})^2}$$

   (a) Starting where we left off in class of Feb. 7 (see classnotes2017-02-07.pdf), derive the dispersion relation and above expressions for phase speed and group velocity. See also my notes from ATS601, pages 46-47.

   (b) These waves owe their existence to the basic state PV gradient. Identify the basic state PV gradient in the dispersion relation and discuss the individual contributions to it. In which way does this basic state PV gradient influence the wave properties (frequency, phase speeds, group velocities)?

   (c) Discuss appropriate limits of small and large horizontal scales. What does the limit of $L_d \to \infty$ correspond to?

   (d) Assume meridional scales $\gg$ zonal scales and show that in this case the stationary wavenumber is given by $k_s^2 = \beta/U$. For atmospheric applications scale the deformation radius as $L_d \sim NH/f_0$ (can you motivate why?) and compare the wavelength of these stationary waves to L_d. What is their group velocity?

(e) Now consider PV on isentropic surfaces and the animation of PV on 330 K for January 2014, provided on the website. The quantitative values of PV are not crucial, but the color shading is such that typical tropospheric values are blue and typical stratosphere values are red. Note that in the tropics the 330 K surface is located near 500 hPa (mid troposphere), but it tilts upward across subtropical latitudes, and is located near 200-250 hPa (lowermost stratosphere) in mid and high latitudes.

Discuss where you can identify Rossby waves and why they only show up strongly at specific latitudes. Based on the observed wave amplitudes – is the linear assumption, which we always use to derive the wave equation, valid?

Focus on the eastern hemispheric wave activity around ∼19-24 January. Identify this wave packet in the longitude-time plot below (this plot shows anomalies from zonal mean). Graphically estimate phase speeds and group velocity, and compare those to the expected values based on above dispersion relation (note, observed zonal wind speeds are ∼35 m/s here). Use $L_d \sim 1000$ km. Discuss.